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# An appointment scheduling framework to balance the production of blood bags from donation

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### An appointment scheduling framework to balance the production of blood bags from donation

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## Abstract.

Blood is fundamental in several care treatments and surgeries, and plays a crucial role in the health care system. It is a limited resource, as it can be produced only by donors and its shelf life is short; thus, the blood donation (BD) system aims at providing adequate supply of blood bags to transfusion centers and hospitals. An effective collection of blood bags from donors is fundamental for adequately feeding the entire BD system and optimizing blood usage. However, despite its relevance, donation scheduling is only marginally addressed in the literature. In this paper we consider the Blood Donation Appointment Scheduling (BDAS) problem, aiming at balancing the production of the different blood types among days in order to provide a quite constant feeding of blood bags to the BD system. We propose a framework for the appointment reservation that accounts for both booked donors and donors arriving without a reservation. It consists of an offline Mixed Integer Linear Programming (MILP) model for preallocating time slots to blood types, and of an online prioritization policy to assign a preallocated slot when the donor calls to make the reservation.

**Keywords**: Blood donation system, Blood donation appointment scheduling, Production balancing, Mixed integer linear programming model, Offline and online procedure.

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#### Abstract

Blood is fundamental in several care treatments and surgeries, and plays a crucial role in the health care system. It is a limited resource, as it can be produced only by donors and its shelf life is short; thus, the blood donation (BD) system aims at providing adequate supply of blood bags to transfusion centers and hospitals. An effective collection of blood bags from donors is fundamental for adequately feeding the entire BD system and optimizing blood usage. However, despite its relevance, donation scheduling is only marginally addressed in the literature. In this paper we consider the Blood Donation Appointment Scheduling (BDAS) problem, aiming at balancing the production of the different blood types among days in order to provide a quite constant feeding of blood bags to the BD system. We propose a framework for the appointment reservation that accounts for both booked donors and donors arriving without a reservation. It consists of an offline Mixed Integer Linear Programming (MILP) model for preallocating time slots to blood types, and of an online prioritization policy to assign a preallocated slot when the donor calls to make the reservation.

#### Keywords

Blood Donation System; Blood Donation Appointment Scheduling; Production Balancing; Mixed Integer Linear Programming Model; Offline and Online Procedure.

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#### 1. Introduction

Blood supply is a key point for all health care systems, as blood is necessary for several care treatments and surgical interventions. For example, in 2012, the annual need for blood was about 10 million units in the USA, 2.1 in Italy, and 2 in Turkey. Blood is also a limited resource because, at present, it cannot be produced in laboratory but only by humans. Thus, in Western countries, blood is usually collected from donors, i.e., unpaid individuals who donate their blood voluntarily. Further, its short shelf life limits the period between donation and utilization, thus preventing long term storage.

Blood is provided through the Blood Donation (BD) system, which is in charge of providing an adequate supply of blood bags to transfusion centres and hospitals. Due to the short shelf life, BD system should meet the overall blood demand from hospitals and transfusion centres, but at the same time it should follow the temporal profile of the demand to avoid blood shortage and wasted bags. The BD supply chain can be divided into four steps, as shown in Figure 1: *collection, transportation, storage* and *utilization* (Sundaram and Santhanam, 2011). Blood is first collected: donors are registered and visited by a physician to assess their eligibility for donation and, if eligible, they make the donation. Once the blood is gathered, tests are performed on each blood bag to prevent infectious diseases. Afterwards, blood bags are transported and stored. Blood components are then distributed to hospitals and transfusion centres based on their inventory levels. Finally, blood is transferred to the end users (the patients) for transfusion.

In this paper, we focus on the blood collection step, which represents the first (and most critical) step of the BD supply chain. Not only increasing the number of donations improves the throughput of the the BD system, but also having an effective management of donors' arrivals among the days may improve the performance of the system and optimize the daily production of blood bags with respect to the demand. On the contrary, an unbalanced feeding of blood bags may undermine the entire BD supply chain and result in alternating periods of blood shortage and wasted bags.

Several blood collection centres are starting to implement a reservation system. In fact, reserving the donation appointment can reduce donors' waiting time and, thus, guarantee a better service to donors, which may help in increasing the number of donors and the frequency of donation. Moreover, by appropriately addressing donors to a suitable day, reservation may also balance the production of blood bags among the days. In any case, centres also accept donors without reservation not to refuse any possible donation, because of the high need for blood bags and to prevent donors from feeling that their donation is not important. Thus, generally speaking, both booked and non-booked donors are usually present in the collection centres, even though the effort is to increase the rate of booked ones. So far, appointments are manually assigned in the majority of collection centres where reservation is possible. Manual management may be able to reduce donors' waiting times



Figure 1: Steps of the BD supply chain.

and to take their preferences into account; however, it is short-sighted and may prevent from effectively balancing blood bag production.

In this work, we propose an appointment scheduling system for blood donation to balance the production of blood bags of the different types (combination of group and Rhesus factor) among days, while taking into account both booked and non-booked donors. The proposed architecture for planning the assignments consists of two phases, i.e., an offline preallocation of time slots for donation and an online allocation of them, where a *time slot* is as an operational or service time interval suitable for a donor. The preallocation phase is responsible of reserving slots to the blood types, while the allocation phase is responsible of assigning a suitable preallocated slot to each donor when he/she calls for reservation. In other words, the preallocation phase prepares a number of spare slots for the different blood types, which are then used for the successive online booking phase. The architecture is based on a Mixed Integer Linear Programming (MILP) model for the preallocation phase and a prioritization policy for the allocation phase.

Although the problem shares some features with other health care related appointment scheduling problems, balancing the production is not a common objective. Moreover, the characteristics of the BD system make the donation scheduling different from other appointment scheduling systems in different fields. Thus, to the best of our knowledge, this paper is the first attempt to deal with what we can define as the Blood Donation Appointment Scheduling (BDAS) problem. In this paper, we particularly consider the case of the Milan Department of the Associazione Volontari Italiani Sangue (AVIS), denoted as AVIS Milan in the following, which can be considered as a general blood collection centre since it shares many features in terms of donors, activities and management with several other centres. Thus, the approach proposed in this paper can be considered as general and applicable to other blood collection centers.

The paper is structured as follows. More details about the BDAS problem are reported in the remainder of this Section. Then, a literature review on BD collection management and appointment scheduling is presented in Section 2. The proposed architecture for the BDAS problem is detailed in Section 3, including the MILP preallocation model and the prioritization policy. An analysis of the MILP preallocation model is further reported in Section 4. Finally, the computational tests performed on the AVIS Milan case and the conclusions are reported in Sections 5 and 6, respectively.

#### 1.1. Problem description

The BD collection phase includes all of the stages between donor's arrival and the complete preparation of the blood bag. The process starts when the donor arrives at the blood collection centre. Here, donors are visited by a physician to assess their eligibility for donation; if eligible, donors make the donation. Once the blood is drawn from an individual, it undergoes a screening process to be searched for any infectious diseases, and the blood bags that pass the tests are sent for storage.

Two main aspects are present in the blood collection step. On the one hand, managing a BD collection centre includes the typical operational problems that are common to several service providers and other health care facilities (e.g., visit centres, hospitals, emergency services). Among them, we mention workforce planning, appointment scheduling, demand prediction, waiting times reduction, service quality improvement, etc. On the other hand, the goal of a BD collection centre is to produce blood bags and blood products to meet the demand from the health care system. Thus, an effective management of a collection centre cannot limit to its internal organization as a service provider, but it must necessarily account for the production of blood bags. An effective management of blood collection is firstly necessary to increase the throughput and keep the costs sustainable. However, a more general view should include an effective management of donors' arrivals throughout the days to optimize the daily production of blood bags with respect to the demand. Neglecting this point may result into an unbalanced feeding of blood bags to the rest of the BD supply chain, with consequent blood shortage and wasted bags. The first point (throughput and costs) is sometimes addressed in the literature; on the contrary, more structured strategies that also include the impact on the whole BD chain are still lacking (see Section 2).

In the practice, appointment scheduling decisions are manually made or supported by short-sighted tools. Even though these tools are able to reduce donor waiting times and physician overtimes, and/or to optimize other operational issues, they do not include any analysis of the daily blood production with respect to the demand while allocating time slots to donors. Hence, the main goal and benefit of a comprehensive scheduling system is to combine these contrasting needs: to improve the operational level while optimizing the produced bags with respect to the demand and its temporal pattern. This is actually the goal of this paper.

From a management point of view, donors can be mainly divided into two groups: *re-turning donors* who donate on a regular basis, and *walk-in donors* who occasionally donate or donate for the first time. In any case, a donation can be made after a rest period from the previous one, which is defined by law. If the blood collection centre has a reservation system, donors can be further classified into *booked* and *non-booked* donors.

As mentioned, we consider the case of AVIS Milan. AVIS was founded in 1927 and nowadays is the largest blood donors association in Italy, bringing together over one million of voluntary blood donors across the country. AVIS Milan covers the territory of Milan and is in charge of collecting blood for one of the main hospitals in Milan, i.e., the Niguarda hospital; in the last 4 years, it provided on average about 50 whole blood donations per day, with a total of about 18000 donations per year.

AVIS Milan is starting to implement a reservation system and currently accepts donors with and without reservation, the second being the majority at the moment, but it aims at increasing the rate of the booked donors. Its goal is to produce a constant amount of blood bags for each blood type due to the fact that the Niguarda hospital is a big hospital with a lot of elective surgeries and a quite constant amount of emergency requests. The lack of a constant feeding to the hospital is the actual bottleneck of the entire system in the practice, based on the discussion with the staff of AVIS Milan. Even some peaks in the demand may occur in specific periods and conditions, the request from Niguarda hospital to AVIS Milan is to feed the system with a constant (and possibly high) daily amount of bags of the different blood types.

The current architecture of the AVIS Milan scheduling system is shown in Figure 2a, which also shares many features with several blood collection centres. Some donors call to book the donation day and time slot beforehand, and slots are assigned (booked) until a maximum percentage of the daily capacity is reached, regardless of blood type. The daily capacity is expressed in terms of the total physician working time without incurring overtime. In fact, while AVIS Milan has a large donation room where a seat is quite always available when a donor arrives, the physician's visit before donation is the bottleneck of the system that generates the queue; thus, we consider the physician working time as the scarce resource and the time slot refers to the time spent for the visit. Some part of the capacity (a maximum percentage) is usually taken into account when a donation is booked, to preserve space for non-booked donors; however, to match the donors' preferences, this threshold can be extended (overpassed) without penalties. The daily donations are finally given by the amount of booked and non-booked donors who show up at the blood collection centre.

Historical data from AVIS Milan show that the number of produced bags is not constant among days. Figure 3a reports the daily number of whole blood bags produced per day, and Figure 3b the relative percentage of bags with type A Rh+ (data refer to 2013 and 2014, i.e., two years in which production balancing was not considered). We can observe that the number of blood bags is not evenly balanced among the days, despite the goal of flattening the production both in terms of total number of bags per day and for the different blood types. In particular, AVIS Milan would like avoiding high frequency oscillations, while low frequency oscillations do not depend on scheduling and cannot be avoided. For example, the decreased production around days 220-240 in Figure 3a corresponds the month of August



Figure 2: Current architecture of AVIS Milan (a) and proposed architecture (b).

when people are usually on holiday and they do not donate.

#### 2. Literature review

In literature, there are two main classifications of the BD supply chain and the related management problems. Sundaram and Santhanam (2011) classify the system based on the main steps of a blood bag life (as mentioned in the Introduction) while, according to Pierskalla (2005), the BD supply chain can be classified based on the strategic and tactical operational decisions.

Many optimization problems are present in managing the BD supply chain, from donation to final utilization of blood bags. Most of them have been largely addressed in the literature, as underlined by recent surveys; e.g., Beliën and Forcé (2012) reviewed the literature up to 2010, and Osorio *et al.* (2015a) presented a structured review on quantitative



Figure 3: Daily number of whole blood donations in 2013 and 2014 according to the historical information of AVIS Milan: total number of donations (a) and percentage of type A Rh+ (b).

modeling for BD supply chain. However, different problems received different attention in the literature and, even though the blood collection step is one of the most important ones at the operational level, the BDAS problem has been never addressed so far. A literature analysis on BD supply chain management conducted by Baş *et al.* (2016) and then updated up to August 2015, which included 177 papers that are available on Scopus and the other main scientific databases, shows that only the 1% of the BD management investigations deal with donor arrival and scheduling.

In the following, we first review the literature dealing with the management of the blood collection step, and then we survey the literature about appointment scheduling systems.

#### 2.1. Blood collection in the literature

Several management problems arise in the blood collection, which can be classified based on the planning (e.g., location of blood collection centres and staff dimensioning) or operational (e.g., appointment scheduling, screening policies, donation prediction) level. Although problems of both levels have an impact on the entire BD chain, problems occurring at the operational level have a direct effect on blood shortages and wasted bags. In the following, we focus on such level, which is closely related to the appointment scheduling system developed in this paper.

Testik et al. (2012) identified donor arrival patterns and employed a queuing network model of the donation process to dimension the workforce. Alfonso et al. (2012, 2013) proposed Petri net models to describe all relevant donor flows in various blood collection systems. Michaels et al. (1993) developed a simulation study to evaluate scheduling strategies for donors who arrive at a Red Cross blood drive, and compared them in terms of mean transit time to find out the most effective one. Mobasher et al. (2015) coordinated appointment and pick-up times at blood donation sites to maximize platelet production. Osorio et al. (2015b) worked on a multi-objective stochastic optimization model for technology selection and donor assignment. Elalouf et al. (2015) improved the structure of a three-echelon blood sample collection chain, which includes clinics, centrifuge centres, and a centralized testing laboratory. More closely to the BDAS problem, Alfonso et al. (2015) presented a simulationoptimization approach for capacity planning and appointment scheduling in blood collection systems, accounting for random service times, random arrivals of walk-in donors, and random no-shows of scheduled donors. The aim is to simoultaneously maximize donor service level and minimize system overtime. However, differently from the BDAS problem, they do not take into account the different blood types and the production balancing, which is instead our main goal.

Stochastic model to predict the waiting time and other random variables are also available. Flegel *et al.* (2000) developed a logistic regression model to compute the donation probability within a given time frame. Ferguson and Bibby (2002) used a prospective design to predict the number of future blood donations. Raven *et al.* (2010) estimated the blood supply from donations using annual donor retention rates and mean numbers of donations per donor and year. Boonyanusith and Jittamai (2012) investigated donor behavior patterns and the factors that influence donation decision. Ritika (2014) found a fair classification technique for donation prediction. Van Dongen *et al.* (2014) analyzed the factors that affect the intention to continue donating in new donors. Van Brummelen *et al.* (2015) developed a model for estimating the waiting time in blood collection sites, which provides the total delay time distribution. Fortsch and Khapalova (2016) proposed a Box-Jenkins method to predict blood demand, aiming at lowering costs and reducing blood wastages.

#### 2.2. Related appointment scheduling systems

Scheduling problems are widely studied in the literature (Gupta and Starr, 2014) and have been classified according to several criteria (e.g., number and sequence of machines, processing times, job arrival rates and objective function) for both manufacturing and service systems, including health care systems.

Effective schedules are widely studied in manufacturing (Pinedo, 2009; Oyetunji, 2009; Sawik, 2011; Pinedo, 2012; Rahman *et al.*, 2015; Jonsson and Ivert, 2015; Han *et al.*, 2015) with the goals of meeting due dates, maximizing machine or labor utilization, and minimizing job lateness, response time, completion time, time in the system, overtime, idle time and work-in-process inventory. A review can be found in Framinan and Ruiz (2010).

Scheduling in service systems is different from that in manufacturing, mainly because the system capacity in manufacturing may exploit inventories. On the contrary, a service is provided together with its utilization; consequently, service capacity cannot be stored and it is lost if unused (Ayvaz and Huh, 2010; Zhou and Zhao, 2010). In service systems, customers want to spend the minimum waiting time and receive good quality service, whereas service providers want to perform the schedule with the minimum cost. In particular, service systems try to satisfy the demand through appointments. Thus, appointment scheduling represents the interface between demand and service provider.

Focusing on health care services, many papers dealing with appointment scheduling are available in the literature (Liu, 2009; Truong, 2015; Wang and Fung, 2015). The goal is usually to maximize the number of patients while minimizing waiting times, physician idle times and overtimes (Gupta and Denton, 2008; Samorani and LaGanga, 2015). Some papers analyze the negative effects of no-shows in terms of provider underutilization and delayed patient access (Robinson and Chen, 2010; Liu and Ziya, 2014; Liu, 2016); in such cases, most of the applied solutions propose overbooking in order to increase the utilization.

The management of the operating theatres has been one of the most studied topics in the last 60 years (Cardoen, 2010; Hans and Vanberkel, 2012). Other widely studied topics are nurse scheduling (Burke *et al.*, 2004; Bai *et al.*, 2010; Lim *et al.*, 2012), patient appointments in ambulatory care (Gupta and Wang, 2012), appointment scheduling in outpatient clinics (Berg and Denton, 2012), bed assignments in hospitals (Hall, 2012), scheduling of urgent patients (Gerchak *et al.*, 1996; Klassen and Rohleder, 2003; Torkki *et al.*, 2006), nurse and surgery scheduling (Beliën and Demeulemeester, 2008), and trade-offs between the cancellation of scheduled elective surgeries to accommodate urgent arrivals (Zonderland *et al.*, 2010).

Blood collection involves both the features of a service system and those of a production system. Thus, the BDAS problem cannot be included by the ordinary classification, and this also explains the lack of BD appointment scheduling systems in the literature.

#### 3. Proposed architecture for the donor appointment scheduling

In this paper we propose a new architecture for the BDAS problem. As mentioned in the Introduction, the proposed architecture for planning the donations consists of two phases, i.e., an offline preallocation of time slots for donation based on the blood type, and an online allocation. The output of the preallocation acts as an input for the allocation, in which the daily layout of prereserved slots is filled while the donors call for booking. Indeed, the allocation phase assigns a preallocated slot to each donor, when he/she calls for reservation, among those prepared in the preallocation phase. Such decomposition in two phases is based on the evidence that, in order to balance the daily production of all blood types, the slots should be assigned in advance to the different types and then the donors should be addressed to the slots of their specific type.

The list of preallocated slots is refreshed (regenerated) after a certain number of reservations are received and/or at a fixed frequency (e.g., each day). The number of preallocated slots which have been converted into reserved slots is fed back to the preallocation phase (the assigned slots are no longer available and have to be considered as occupied) and the process is repeated. As a result, the plan for each day is given by the list of booked donors for that day, together with the number of empty slots that are left free for the non-booked donors who may arise to donate.

Besides the goal of production balancing, the daily layout of prereserved slots should meet some other requirements: the total number of slots should be around the expected number of donors, the slots should respect the proportions of the blood types, and an appropriate number of spare slots should be preserved for non-booked donors. To meet these requirements, the future amount of donors (both booked and non-booked) is required and should be predicted, e.g., based on the available historical data.

The proposed architecture is summarized in Figure 2b. The preallocation phase receives the expected number of booked and non-booked donors, together with the number of occupied (already booked) slots, and provides the preallocated slots  $x_t^b$  (i.e., number of preallocated slots for blood type b at day t of the time horizon). Then, the allocation phase uses these preallocated slots to respond to the phone calls for reservation, and updates the list of occupied slots.

As mentioned before, the preallocation phase is based on a MILP model whereas the allocation phase on a prioritization policy of the prereserved slots. They are detailed in the next two subsections.

#### 3.1. Optimization model for the preallocation of slots

The preallocation of the slots is optimized through a MILP model, whose aim is to preallocate a balanced number of slots for each blood type close to the expected number of booked donors in the considered time horizon. While doing so, some spare time slots are left for non-booked donors, and physician overtimes are penalized.

A set of days T represents the considered time horizon, and all days  $t \in T$  are divided into a set K of periods. Moreover, the set of blood types is denoted as B. We consider for each day t and each blood type b a number of slots  $x_t^b$  to preallocate (non-negative integer decision variable) and a number of already allocated slots  $a_t^b$  coming from previous reservations (integer parameter).

We assume an expected number  $d_b$  of booked donors for blood type b over T. Thus, ideally, the summation over T of the already booked slots and of the slots to preallocate should equal to this value for each blood type, i.e.,  $\sum_{t \in T} (x_t^b + a_t^b) = d_b$ . However, as mentioned, we cannot know this value in advance and, so we assume that each  $d_b$  is affected

	$\mathbf{Sets}$						
В	set of blood types						
T	time horizon						
K	set of time periods $\forall t \in T$						
	Parameters						
$d_b$	expected number of booked donors over $T$ with blood type $b$						
ε	uncertainty associated with $d_b$ (same $\forall b \in B$ )						
$a_t^b$	number of already booked donors at day $t$ with blood type $b$						
$n_t^b$	expected number of non-booked donors at day $t$ with blood type $b$						
$\alpha_k$	fraction of $n_t^b$ in period k (same $\forall t \in T$ )						
$c_{tk}$	overall capacity of physicians (time) in period $k$ of day $t$						
r	standard time required for visiting a donor						
$R_{tk}$	time amount for visiting the already booked donors in period $k$ of day $t$						
$\eta$	maximum variation weight (for the objective function)						
$\delta_k$	penalty for overtime in period k (same $\forall t \in T$ , for the objective function)						
Decision variables							
$x_t^b$	number of preallocated slots for blood type $b$ in day $t$						
$w_{tk}^b$	number of preallocated slots for blood type $b$ in period $k$ of day $t$						
$y_t^b$	number of planned bags for blood type $b$ in day $t$						
$z_t^b$	absolute variation of $y_t^b$ with respect its average value over $T$						
v	maximum of the variations $z_t^b \ \forall t \in T, b \in B$						
$p_{tk}$	physicians' overtime in period $k$ of day $t$						

Table 1: Sets, parameters and decision variables for the preallocation model.

by uncertainty. To this end, we formalize the uncertainty by imposing that the summation  $\sum_{t\in T} (x_t^b + a_t^b)$  can lay in the interval from  $(1 - \varepsilon) d_b$  to  $(1 + \varepsilon) d_b$  for each blood type b, where  $\varepsilon$  is an index of the associated uncertainty. In case of low uncertainty, a small  $\varepsilon$  value close to 0 can be assumed, whereas higher values up to 1 can be taken in case of highly uncertain donor arrivals. Forcing the system to allocate a given number of slots (actually a number in a range) is necessary in the presence of an objective function that aims at balancing the production of bags among days and avoiding overtimes. In fact, on the one hand, a perfect balancing with no overtime can be obtained with a null production. On the other hand, preallocating a number of slots higher than the necessary amount will lead to several empty slots because of fewer calls for reservation; thus, even though the preallocated slots are balanced, the actually occupied slots could be unbalanced. Hence, an appropriate selection of  $\varepsilon$  value is crucial.

As indicated, an expected amount of slots should be left empty for non-booked donors,

which is represented by  $n_t^b$  for blood type b and day t. Since non-booked donors may arrive in any period k of the day  $(k \in K)$ , the fraction of  $n_t^b$  for period k is denoted with  $\alpha_k$  (we assume the same division  $\forall t \in T$ ).

The overall capacity of the physicians in period k of day t, without incurring overtime, is denoted by  $c_{tk}$ . The standard time r required for visiting a donor (considered while allocating new slots  $x_t^b$ ) is assumed to be constant and equal for all donors. In addition, for the already booked slots  $a_t^b$ , a specific service duration can be set for each donor; we denote by  $R_{tk}$  the total time for the already allocated donors in period k of day t. Note that, at each day t, the number of already allocated slots  $a_t^b$  are grouped by blood type b, while the associated times  $R_{tk}$  are grouped by period k.

Some additional decision variables are included to model the preallocation problem. The number of preallocated slots for blood type b in day t and period k is represented by an non-negative integer variable  $w_{tk}^b$ , whose sum over  $k \in K$  provides  $x_t^b$ . The overall number of planned donations for blood type b at day t is  $y_t^b$ , which is given by  $x_t^b + a_t^b + n_t^b$ . The absolute variation of  $y_t^b$  with respect to its average value over the days t is denoted as  $z_t^b$ . Finally, the overtime required above the capacity  $c_{tk}$  at day t and period k is denoted by  $p_{tk}$ .

Sets, parameters and decision variables are summarized in Table 1. Variables are subject to the following constraints:

$$y_t^b = x_t^b + n_t^b + a_t^b, \qquad \forall t \in T, b \in B$$

$$(1)$$

$$\sum_{\tau \in T} y^o_\tau - y^o_t |T| \le z^o_t |T|, \qquad \forall t \in T, b \in B$$
(2)

$$y_t^b|T| - \sum_{\tau \in T} y_\tau^b \le z_t^b|T|, \qquad \forall t \in T, b \in B$$
(3)

$$v \ge z_t^b, \qquad \forall t \in T, b \in B$$
 (4)

$$(1-\varepsilon) d_b \le \sum_{t \in T} \left( x_t^b + a_t^b \right), \qquad \forall b \in B$$
(5)

$$\sum_{t \in T} \left( x_t^b + a_t^b \right) \le (1 + \varepsilon) \, d_b, \qquad \forall b \in B$$
(6)

$$x_t^b = \sum_{k \in K} w_{tk}^b, \qquad \forall t \in T, b \in B$$
(7)

$$r\sum_{b\in B} \left(w_{tk}^b + \alpha_k n_t^b\right) + R_{tk} \le c_{tk} + p_{tk}, \qquad \forall k \in K, t \in T$$
(8)

$$\begin{aligned} x_t^* &\geq 0, x_t^* \in \mathbb{N}, & \forall t \in T, b \in B \\ y_t^b &\geq 0, y_t^b \in \mathbb{N}, & \forall t \in T, b \in B \\ p_{tk} &\geq 0, & \forall t \in T, k \in K \\ w_{tk}^b &\geq 0, w_{tk}^b \in \mathbb{N}, & \forall k \in K, t \in T, b \in B \end{aligned}$$

Constraints (1) compute the number of blood bags  $y_t^b$  for each day t and blood type b. Constraints (2) and (3) calculate the absolute variation  $z_t^b$  between  $y_t^b$  and its average value over T, and constraints (4) compute the maximum of such absolute variations. Constraints (5) and (6) force the total number of slots of type b to be around  $d_b$ , with tolerance  $\varepsilon$ ; obviously, the number of slots is an integer number, so that the effect of these constraints is to bound  $\sum_{t \in T} (x_t^b + a_t^b)$  between  $\lceil (1 - \varepsilon) d_b \rceil$  and  $\lfloor (1 + \varepsilon) d_b \rfloor$ . Constraints (7) calculate, for each blood type b, the total number of preallocated slots  $x_t^b$  in day t based on the  $w_{tk}^b$  amounts. Constraints (8) calculate the overtime  $p_{tk}$  based on the times for visit and physicians' capacity.

In this formulation, we assume that all arriving donors make a donation, that all booked donors show up at the right period and day, and we do not consider different types of donations other than the whole blood donation (e.g., apheresis).

The primary objective of the model is to balance the production of each blood type b among the days, which corresponds to obtaining low  $z_t^b$  values. Moreover, the secondary goal is to minimize the physicians' overtimes  $p_{tk}$ , where the overtime of each period  $k \in K$  is penalized through a specific weight parameter  $\delta_k$ . Hence, the following objective function is considered, which is composed by three terms:

$$\min\left\{\sum_{b\in B}\sum_{t\in T} z_t^b + \eta v |T||B| + \sum_{t\in T}\sum_{k\in K_t} \delta_k p_{tk}\right\}$$
(9)

The first two terms (named OF1 and OF2, respectively) balance the production among days by reducing the absolute variations  $z_t^b$ ; OF1 minimizes the total absolute variation with respect to the average production, while OF2 minimizes the maximum absolute variation among all days and all blood types. The third term (named OF3) minimizes the total weighted physicians' overtimes. The three terms may be optimized all together, as reported in (9), or alternatively we can consider only one or two of them. If OF2 is neglected, constraints (4) can be removed from the model, whereas constraints (8) can be removed if OF3 is not considered.

Let us focus on the first two terms OF1 and OF2, which both aim at balancing the production.  $\eta$  is a positive parameter that represents the relative weight of the maximum absolute variation with respect to the total one: a low value of  $\eta$  favors the total variation, whereas higher values favor the maximum variation. Parameter v is multiplied by |T| and |B| to obtain, with  $\eta = 1$ , the same order of magnitude for the two terms. It is common in optimization problems that both the summation and the maximum of a set of decision variables are optimized. But, in our case, these two terms may lead to allocate a different number of slots  $x_t^b$ , since  $y_t^b$  is given by  $x_t^b + n_t^b + a_t^b$  and the summation  $\sum_{t \in T} x_t^b + a_t^b$  is not constrained to a value but to a range, due to (5) and (6). On the contrary, in several other problems, the overall amount is generally fixed and just differently allocated. Further details will be provided in Section 4.

#### 3.2. Prioritization policy for the online allocation of slots

The goal of the prioritization policy is to decide the best preallocated slot to propose when a donor calls to make a reservation. However, proposing only one day to the donor is not enough because the donor may have other constraints and could not accept the proposal. Thus, it is preferable to propose a list of possible days t and periods k, and let the donor choose among them. This might increase the donation frequency and the perceived usefulness of the donation from the donor. Hence, the goal of this second phase is to assign a score to each slot of the donor's blood type, such that the slots can be proposed one by one to the donor in a decreasing order of score until a slot is accepted. This is a good compromise between donor's needs (propose several alternatives) and production needs (propose the best alternative).

Basically, there are two points behind the prioritization of the slots and the assignment of the score: to fill the first available day and to keep the flexibility of the reservation system. The first point requires assigning the donor in the first available day according to his/her blood type. In fact, keeping the first available slots empty may negatively affect the system if no further donors of the same blood type will ask for reserving a donation, because such slots will remain empty. The second point requires not to fill all of the preassigned slots of a day; otherwise, the range of choice for the next calling donor is reduced. Hence, flexibility means to assign donors in the day with the highest number of preallocated slots still available. Both points are taken into account while assigning scores, each one weighted by a value. The score  $S_{tkb}$  of slots  $w_{tk}^b$  is computed ( $\forall t, k, b$ ) by the following linear formula:

$$S_{tkb} = \lambda_f w_{tk}^b - \lambda_d t \tag{10}$$

where t represents, according to the MILP model, the day in the time horizon, starting from the current one in which reservations are arriving (t = 1).

The first term generates higher scores for higher values of  $w_{tk}^b$ , i.e., when the flexibility remains higher if the donor of blood type b is allocated to t and k; the second term, due to the minus sign, generates higher scores when the donor is allocated to as low as possible values of t (i.e., to a closer day).  $\lambda_f$  is the weight of the flexibility term, while  $\lambda_d$  is the weight of the early allocation term.

Preallocated time slots are thus sorted and proposed one by one in a decreasing order of score. If the donors accepts the first proposed slot, this maximizes the goals of the system. In any case, we remark that every request for reservation is accepted: if no slots are available in the donors suitable days, an additional slot is forced with respect of the preallocated ones.

#### 4. Subproblems and valid inequalities

In this section we analyze some subproblems, to show the different behaviors of OF1 and OF2, and to derive valid inequalities that could speed up the computational times.

#### Subproblem 1

Let us consider the case of one blood type  $b^*$  alone (|B| = 1), no preallocated slots  $(a_t^{b^*} = 0, \forall t)$ , a constant number of non-booked donors  $(n_t^{b^*} = \bar{n}^{b^*}, \forall t)$ , and infinite capacities  $(c_{tk} \to \infty, \forall t, k)$ . The range for  $M = \sum_{t \in T} x_t^{b^*} + a_t^{b^*} = \sum_{t \in T} x_t^{b^*}$  is constrained between  $M_{min} = \lceil (1 - \varepsilon) d_{b^*} \rceil$  and  $M_{max} = \lfloor (1 + \varepsilon) d_{b^*} \rfloor$  by (5) and (6).

If there exists a multiple of |T| in  $[M_{min}, M_{max}]$ , then a perfect balancing with  $z_t^{b^*} = 0$ ,  $\forall t \in T$  is possible. Otherwise, the best possible balancing is given by allocating blocks of |T| time slots (one slot for each day  $t \in T$ ) until the remaining number of slots to allocate is lower than |T|. This remaining number  $N = M - |T| \left\lfloor \frac{M}{|T|} \right\rfloor$  (with 0 < N < |T|) is responsible of an unavoidable unbalancing because, at the optimum, N slots are allocated in N days (one for each day), while no slots are allocated in the other |T| - N days. Coherently,  $z_t^{b^*} = 1 - \frac{N}{|T|}$  in the N days in which a remaining slot is allocated, while  $z_t^b = \frac{N}{|T|}$  in the |T| - N days in which no remaining slots are allocated. Thus:

$$\sum_{t \in T} z_t^{b^*} = N\left(1 - \frac{N}{|T|}\right) + (|T| - N)\frac{N}{|T|} = 2\left(N - \frac{N^2}{|T|}\right);$$
(11)

$$v = \max\left\{z_t^{b^*}, t \in T\right\} = \begin{cases} 0 & N = \{0; |T|\}\\ \max\left\{1 - \frac{N}{|T|}; \frac{N}{|T|}\right\} & N \in [1, |T| - 1]. \end{cases}$$
(12)

The first expression in (11) is a parabola with maximum in  $N = \frac{|T|}{2}$  and null value in N = 0and N = |T|. The second expression in (12) assumes a null value for N = 0 and N = |T|, while for  $N \in [1, |T| - 1]$  it is a V-shaped function with minimum value 0.5 in  $N = \frac{|T|}{2}$ .

The remaining number N must be chosen within the range  $[N_{min}, N_{max}]$ , where  $N_{min}$  and  $N_{max}$  are the remaining parts of  $M_{min}$  and  $M_{max}$ , respectively, which are defined as follows:

$$N_{min} = M_{min} - |T| \left\lfloor \frac{M_{min}}{|T|} \right\rfloor$$
$$N_{max} = M_{max} - |T| \left\lfloor \frac{M_{min}}{|T|} \right\rfloor$$

This degree of freedom is responsible of the different behaviors between OF1 and OF2 in terms of allocated  $x_t^b$ . By constraining the domain of N to  $[N_{min}, N_{max}]$ , the minimum of (11) is in the farthest point from the maximum of the parabola, i.e., in  $N_{min}$  if  $N_{min} < |T| - N_{max}$ , or in  $N_{max}$  if  $N_{min} > |T| - N_{max}$ . As a consequence, OF1 prefers to allocate a number of M slots as close as possible to a multiple of |T|. On the contrary, if a perfect balancing is not possible, the minimum of (12) is obtained by allocating a number of slots M as close as possible to the intermediate value between two consecutive multiples of |T|.

Intermediate behaviors can be obtained when both OF1 and OF2 are present, which can be adjusted by varying the relative weight  $\eta$ .

#### Subproblem 2

Let consider again one blood type  $b^*$  alone (|B| = 1), a constant number of non-booked donors  $(n_t^{b^*} = \bar{n}^{b^*}, \forall t)$ , and infinite capacities  $(c_{tk} \to \infty, \forall t, k)$ . But, now, let consider some preallocated slots  $a_t^{b^*}$ . Two cases may occur:

- Subproblem 2a: if  $a_t^{b^*} \leq \xi_t^{b^*} \forall t$  (where  $\xi_t^{b^*}$  denotes the optimal value of  $x_t^{b^*}$  in the corresponding Subproblem 1 where  $a_t^{b^*} = 0$ ) the same considerations derived for Subproblem 1 still hold, and (11) and (12) are valid. Indeed, slots either belong to  $a_t^{b^*}$  or  $x_t^{b^*}$ , but the constraint on the summation  $\sum_{t \in T} (x_t^{b^*} + a_t^{b^*})$  acts in the same way and the same values of OF1 and OF2 are reached.
- Subproblem 2b: if  $\exists \tilde{t} : a_{\tilde{t}}^{b^*} > \xi_{\tilde{t}}^{b^*}$ , it is not possible to reach the same balancing of Subproblem 2a, and higher values of OF1 and OF2 are obtained. Even though we allocate the slots  $x_{t}^{b^*}$  in a balanced way with  $x_{\tilde{t}}^{b^*} = 0$ , the higher value of  $y_{\tilde{t}}^{b^*}$  with respect to the mean daily production remains, thus giving an unbalanced solution. In this subproblem, we cannot derive an analytical expression as for Subproblem 1, but the best possible balancing can be derived with an algorithm (which is out the scope of this paper).

#### Subproblem 3

Let consider again infinite capacities  $(c_{tk} \to \infty, \forall t, k)$  but more than one blood type, i.e., |B| > 1. Due to the presence of unlimited capacity without overtime, the problem can be decomposed by balancing the blood types individually. Hence, for each blood type  $b \in B$ , a Subproblem 1 or Subproblem 2 can be considered.

#### Other problems

To move towards the complete problem, we can remove the assumption of infinite capacities (while also including OF3 in the objective function) or we can consider a variable amount of non-booked donors  $n_t^b$  among days t. In the most general case, both these aspects can be included.

By removing the assumption of infinite capacity, the slots of the different blood types cannot be preallocated individually, and we cannot decompose the problem anymore. Due to the competing blood types and the resulting overtime costs, the best balancing previously obtained with *Subproblem 1* or *Subproblem 2a* could not be achieved. Indeed, while improving the balancing, the additional overtime cost in OF3 could be more expensive than the corresponding reduction of OF1 and/or OF2, and the system would prefer more unbalanced solutions.

As for variable  $n_t^b$  values, an expression for the best possible balancing can be derived while considering blood types individually, but a close analytical formula does not exist and an algorithm is required, as for the  $a_{\tilde{t}}^{b^*} > \xi_{\tilde{t}}^{b^*}$  case of *Subproblem 2*.

#### Lower bounds

In case of unavoidable unbalancing, it can be time consuming to close the gap between the integer solution and the continuous relaxation in the commercial solvers (e.g., CPLEX solver). Indeed, the continuous relaxation splits N among the days with fractional allocations, whereas the actual integer solution does not. Thus, the branch-and-bound procedure continues, systematically generating sub problems to analyze and discarding those that do not improve the objective lower bound. To this end, valid inequalities can be added to reduce computational times, i.e., additional cuts that reduce the admissible region of only the continuous relaxation by bounding the values of OF1 and OF2.

In case of constant  $n_t^b$ , we bound OF1 and OF2 with the best possible balancing obtained for *Subproblem 1* and *Subproblem 2a*, which correspond to the minimum of (11) and (12), respectively.

The lowest value of OF1 for a given blood type  $b^*$  is given by (see *Subproblem 1*):

$$2\min\left\{N_{min} - \frac{N_{min}^2}{|T|}; N_{max} - \frac{N_{max}^2}{|T|}\right\}$$

Its summation over the blood types, assuming a null value for those types where a perfect balancing is possible, gives the lower bound  $LB_{OF1}$ . Hence, the following lower bound constraint LB1 is added to the model:

$$\sum_{b \in B} \sum_{t \in T} z_t^b \ge LB_{OF1} \tag{13}$$

We remark that  $LB_{OF1}$  is computed from the available data before the model is run and, thus, it is another model parameter.

The lowest value of OF2 for a given blood type  $b^*$  is given, according to (12), by:

$$\begin{cases} 0 & N_{min} = 0 \text{ or } N_{max} = |T| \\ 1 - \frac{N_{max}}{|T|} & N_{min} > 0 \text{ and } N_{max} \le \frac{|T|}{2} \\ \frac{N_{min}}{|T|} & N_{min} \ge \frac{|T|}{2} \text{ and } N_{max} < |T| \\ \frac{1}{2} & 0 < N_{min} < \frac{|T|}{2} \text{ and } \frac{|T|}{2} < N_{max} < |T| \end{cases}$$

Then, the highest of the values among the blood types b gives the lower bound  $LB_{OF2}$ . Hence, the following lower bound constraint LB2 is added to the model:

$$v \ge LB_{OF2} \tag{14}$$

We remark that also  $LB_{OF2}$  is computed from the available data and it is another model parameter.

Due to the opposite behaviors of OF1 and OF2, the lower bounds  $LB_{OF1}$  and  $LB_{OF2}$  cannot be reached at the same time (when greater than 0), and the lower bound of their

summation is for sure higher than  $LB_{OF1} + \eta |T||B|LB_{OF2}$ . Another constraint could be introduced to bound such summation; however, no closed formula are possible in this case. We should compute  $\sum_{b \in B} \sum_{t \in T} z_t^b + \eta v |T||B|$  for each possible combination of the values Nof the different blood types, between their respective  $N_{min}$  and  $N_{max}$ ; the minimum of the computed values is the lower bound for the summation.

In case of variable  $n_t^b$ , the absolute variations  $z_t^b$  also depend on the temporal patterns of  $n_t^b$ , and the lower bounds cannot be computed by exploiting (11) and (12). They can be again computed for individual blood types with simple algorithms that search for the most balanced pattern  $\{y_t^{b^*}, t \in T\}$  among the possible ones (given  $n_t^{b^*}$ ,  $M_{min}$  and  $M_{max}$ ); however, in this paper we only focus on lower bounds that can be analytically expressed.

#### 5. Computational tests

In this section we present the computational tests run to analyze the behavior of the preallocation model, and to evaluate the performance of the proposed approach (preallocation model and prioritization policy) over a period of time in a realistic scenario. We first show the results from the preallocation model, considering the impact of the modeling assumptions and the related parameters (Section 5.1), and the computational aspects (Section 5.2). Finally, we present the outcomes of the entire approach in Section 5.3.

The preallocation model is implemented in IBM ILOG OPL and solved via CPLEX 12. The entire approach is implemented in Microsoft Visual Basic, and the developed solution integrates the data and the prioritization policy with the input and the output of the OPL model. All experiments are run on a Windows Machine installed on a server with CPU Intel<sup>®</sup> Core<sup>TM</sup> i3, 2.40 GHz, and 4 GB of dedicated RAM.

#### 5.1. Modeling assumptions and parameters

In this section we test the behavior of the model in response to different parameter values, to analyze both the impact such different values and of our modeling assumptions (e.g., the impact of modeling uncertainty through  $\varepsilon$ ).

Tested instances are divided into two groups, namely A.1 and A.2. Group A.1 includes *balanced* instances, where balanced refers to constant amounts of non-booked donors  $(n_t^b)$  over the planning horizon t for each blood type b, while group A.2 includes the *unbalanced* instances. In both groups we further consider three levels for the fraction of non-booked donors with respect to the total number of donors: Low (L), Medium (M), and High (H). The list of the instances is reported in Table 2. Note that in all cases, for the sake of simplicity, booked donors are not considered  $(a_t^b = 0, \forall t, b)$ .

All instances are generated by considering 8 blood types (|B| = 8), 7 days of time horizon (|T| = 7) with 3 periods (|K| = 3), and capacities  $c_{tk}$  equal to 240, 300 and 180 minutes for

Group	Non-booked level	$d_b, \forall b$	$\sum_t n_t^b,  \forall b$
	Low $(L)$	51	0
A.1	Medium $(M)$	34	17
	High (H)	17	34
	Low $(L)$	51	0
A.2	Medium $(M)$	34	17
	High (H)	17	34

Table 2: Summary of the instances.

k = 1, 2, 3, respectively, in all days t. Visit durations are assumed to be 15 minutes (r = 15) and  $\alpha_k$  fractions are considered equal to 0.5, 0.3 and 0.2 for k = 1, 2, 3, respectively.

Several experiments are conducted by varying  $\varepsilon$  and  $\delta_k$  values, i.e., the parameters that are related to the main assumptions of the proposed model:  $\varepsilon$  deals with the uncertainty of  $d_b$ , and  $\delta_k$  weights the overtime (OF3) with respect to the production balancing (OF1 and OF2). A time limit of 5400 seconds has been imposed in all experiments.

For each instance group and level of non-booked donors, 20 different combinations of  $\varepsilon$ and  $\delta_k$  values are tested, while fixing  $\eta = 1$  and considering the entire objective function (OF1 + OF2 + OF3). Results are reported in Tables 3 and 4. It can be observed that, for higher  $\delta_k$  values (i.e.,  $\delta_k = \{8; 6; 3\}$  and  $\delta_k = \{0.8; 0.6; 0.3\}$ ), the overtime term OF3 is privileged, with consequent higher OF1 and OF2 values (meaning an unbalanced system) for lower  $\varepsilon$  values. For higher  $\varepsilon$  values, the system remains balanced also with high  $\delta_k$  values, because of the flexibility given by the larger range around  $d_b$ . On the other hand, lower  $\delta_k$  values (i.e.,  $\delta_k = \{0.08; 0.06; 0.03\}$  and  $\delta_k = \{0.008; 0.006; 0.003\}$ ) result in completely balanced solutions as soon as  $\varepsilon > 0$ , which also show decreasing OF3 values while increasing  $\varepsilon$ . Only for Level H of Group A.2, the high unbalanced arrival of non-booked donors always prevent from a perfect balacing (OF1 $\neq 0$  and OF2 $\neq 0$ ,  $\forall \varepsilon$ ) and determines increasing OF3 values with  $\varepsilon$ , because the system tries to compensate the unbalancing by adding slots.

These trends are confirmed by the ratio between the number of allocated slots  $(\sum_t \sum_b x_t^b)$ and the total number of expected donors  $(\sum_b d_b)$ . Figure 4 shows this ratio as a function of  $\varepsilon$  for both gropus and all levels of non-booked donors (for  $\delta_k = \{0.08; 0.06; 0.03\}$ ). It can be seen that, except in level H of Group A.2, the number of allocated slots decreases while  $\varepsilon$  increases. Thus, the observed better balancing and lower overtimes for higher  $\varepsilon$  vales are due to the reduced number of assigned slots. On the contrary, as for level H of Group A.2, the model allocates more slots to partially compensate the unbalancing given by the high and unbalanced amount of non-booked donors.

06; 0.003	OF3	3.24	0.00	0.00	0.00	0.00	3.25	2.53	0.00	0.00	0.00	$5.06^{*}$	4.34	4.34	2.90	2.90
0.008; 0.00	OF2	40.00	0.00	0.00	0.00	0.00	40.00	0.00	0.00	0.00	0.00	$40.00^{*}$	0.00	0.00	0.00	0.00
$\delta_k = \{0$	OF1	22.86	0.00	0.00	0.00	0.00	22.86	0.00	0.00	0.00	0.00	$22.86^{*}$	0.00	0.00	0.00	0.00
0; 0.03	OF3	32.40	0.00	0.00	0.00	0.00	32.49	25.29	0.00	0.00	0.00	$50.58^{\circ}$	43.38	43.38	28.98	28.98
0.08; 0.06	OF2	40.00	0.00	0.00	0.00	0.00	40.00	0.00	0.00	0.00	0.00	$40.00^{\circ}$	0.00	0.00	0.00	0.00
$\delta_k = \{$	OF1	22.86	0.00	0.00	0.00	0.00	22.86	0.00	0.00	0.00	0.00	$22.86^{\bullet}$	0.00	0.00	0.00	0.00
$; 0.3 \}$	OF3	324.00	0.00	0.00	0.00	0.00	324.90	36.90	0.00	0.00	0.00	505.80	$361.80^{*}$	$289.80^{*}$	289.80	289.80
= {0.8;0.6	OF2	40.00	0.00	0.00	0.00	0.00	40.00	48.00	0.00	0.00	0.00	40.00	$40.00^{*}$	$32.00^{*}$	0.00	0.00
$\delta_k =$	OF1	22.86	0.00	0.00	0.00	0.00	22.86	13.71	0.00	0.00	0.00	22.86	$22.86^{*}$	$27.43^{*}$	0.00	0.00
$;3\}$	OF3	3240.00	0.00	0.00	0.00	0.00	3249.00	369.00	0.00	0.00	0.00	5058.00	3618.00	$2898.00^{*}$	2898.00	2898.00
$k = \{8; 6\}$	OF2	40.00	0.00	0.00	0.00	0.00	40.00	48.00	0.00	0.00	0.00	40.00	40.00	$32.00^{*}$	0.00	0.00
δ	OF1	22.86	0.00	0.00	0.00	0.00	22.86	13.71	0.00	0.00	0.00	22.86	22.86	$27.43^{*}$	0.00	0.00
	ω	0.00	0.25	0.50	0.75	1.00	0.00	0.25	0.50	0.75	1.00	0.00	0.25	0.50	0.75	1.00
Non-booked	level	Low					Medium					High				

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pooked		-0	$\mathfrak{h}_k = \{8; 6;$	3}	$\delta_k =$	$\{0.8; 0.6$	$; 0.3 \}$	$\delta_k = 4$	(0.08; 0.06)	$; 0.03 \}$	$\delta_k = \{0.$	008; 0.006	$; 0.003 \}$
	ω	OF1	OF2	OF3	OF1	OF2	OF3	OF1	OF2	OF3	OF1	OF2	OF3
	00.C	22.86	40.00	3240.00	22.86	40.00	324.00	22.86	40.00	32.40	22.86	40.00	3.24
0	0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0	0.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
m (	00.C	22.86	40.00	3666.00	22.86	40.00	366.60	22.86	40.00	36.66	22.86	40.00	3.67
0	0.25	13.71	48.00	786.00	13.71	48.00	78.60	$0.00^{*}$	$0.00^{*}$	$29.46^{*}$	0.00	0.00	2.94
0	0.50	0.00	0.00	480.00	0.00	0.00	48.00	0.00	0.00	4.80	0.00	0.00	0.48
0	0.75	0.00	0.00	480.00	0.00	0.00	48.00	0.00	0.00	4.80	0.00	0.00	0.48
	1.00	0.00	0.00	480.00	0.00	0.00	48.00	0.00	0.00	4.80	0.00	0.00	0.48
0	00.C	54.86	152.00	6402.00	54.86	152.00	640.20	54.86	152.00	64.02	54.86	152.00	6.40
0	0.25	$69.14^{*}$	$184.00^{*}$	$5304.00^{*}$	68.57	176.00	533.10	$41.14^{*}$	$120.00^{*}$	78.42*	$41.14^{*}$	$120.00^{*}$	$7.84^{*}$
0	0.50	$69.14^{*}$	$184.00^{*}$	$5304.00^{*}$	68.57	176.00	533.10	27.43	96.00	89.22	34.29	88.00	9.28
0	0.75	$69.14^{*}$	$184.00^{*}$	$5304.00^{*}$	68.57	176.00	533.10	16.00	56.00	107.22	16.00	56.00	10.72
	1.00	$69.14^{*}$	$184.00^{*}$	$5304.00^{*}$	68.57	176.00	533.10	$13.71^{*}$	$48.00^{*}$	$110.82^{*}$	$27.43^{*}$	$32.00^{*}$	$11.80^{*}$

Table 4: Impact of  $\varepsilon$  and  $\delta_k$  on the objective function terms for Group A.2; \* indicates that the run is terminated because the memory limit has been reached.



Figure 4: Allocated slots  $(\sum_t \sum_b x_t^b)$  over demand  $(\sum_b d_b)$  for 5 different  $\varepsilon$  values and 3 non-booked donor levels: low (L), medium (M) and high (H). Subfigure (a) refers to Group A.1 and subfigure (b) to Group A.2.

Other analyses are conducted to investigate how overtime is divided among periods  $k \in K$  (Figure 5), how booked and non-booked donors are scheduled in a day (Figure 6), and how many bags per day are produced while trying to balance the production (Figure 7). All figures refer to the case with all terms in the objective function (OF1, OF2 and OF3), and with parameters  $\delta_k = \{0.08; 0.06; 0.03\}, \eta = 1$  and  $\varepsilon = 0.25$ .

Figure 5 shows the average utilization among days t for each period k, where utilization in t and k is given by  $\left(r \sum_{b \in B} \left(w_{tk}^b + \alpha_k n_t^b\right) + R_{tk}\right) / c_{tk}$ , and for the 3 levels of non-booked donors (L, M and H). In general, results show the possibility of shifting the overtime to the period k with the lowest penalty  $\delta_k$ . However, for level H, overtime is also present in the most penalized period of the day (i.e., k = 1 in our case) because of the high and unbalanced number of non-booked donors, which are not controllable.

Figure 6 shows the daily workload (compared with the capacity) for each day t. It can be seen that the model equally divides the total workload among days, as production balancing is the primary objective. Moreover, equal proportions of booked and non-booked donors are found in all days for group A.1, while the proportions vary from day to day in group A.2. This means that, in the presence of balanced non-booked donors, the system equally allocates slots in the days to keep the situation balanced, while slots are preallocated to compensate the unbalanced input in the presence of unbalanced non-booked donors.



Figure 5: Average utilization for 3 different periods, namely early morning (k = 1), late morning (k = 2), and afternoon (k = 3). Subfigure (a) refers to Group A.1 and subfigure (b) to Group A.2, both including the 3 levels of non-booked donors.



Figure 6: Daily workload for the 3 levels of non-booked donor: L (first column in each day), M (second column in each day), and H (third column in each day). Subfigure (a) refers to Group A.1 and subfigure (b) to Group A.2.



Figure 7: Minimum, average and maximum daily production for a blood type (same values for all types) for the 3 levels of non-booked donors: low (L), medium (M) and high (H). Subfigure (a) refers to Group A.1 and subfigure (b) to Group A.2.

Lastly, Figure 7 shows the minimum, average and maximum daily production of blood bags among days t for a given blood type b (values are the same for all blood types, as the same  $d_b$  values are used  $\forall b$ ). The model perfectly balances the daily production; the only difference is again due to the unbalanced number of non-booked donors that affects the production in the H case of group A.2.

It can be seen from the analyses that the amount of non-booked donors, in the presence of unbalanced arrivals, has a great impact on the system, both in terms of overtime and balancing (see in particular level H of Group A.2). However, with an appropriate set of parameters, the model is able to find a good trade-off between production balancing and overtime reduction also in this case. Thus, the decision maker can choose the preferred set of parameters based on his/her priorities and the features of the blood collection centre. Another parameter with a high impact is  $\varepsilon$ , which models the uncertainty degree associated with  $d_b$ . As shown, high values of  $\varepsilon$  may deteriorate the quality of the solution and, in particular, reduce the amount of produced bags. Thus, the decision maker should accurately set this value not to constrain the solution on a number of donors different from the actual one, but also not to reduce the production without a real motivation coming from the data.

Instance	T	ε	$a_t^b$
I.1	14	0.1	0
I.2	14	0.1	40-80%
I.3	14	0.25	0
I.4	14	0.25	40-80%
I.5	28	0.1	0
I.6	28	0.1	40-80%
I.7	28	0.25	0
I.8	28	0.25	40-80%

Table 5: Instances for the analysis of computational times and LBs.

#### 5.2. Computational times and lower bounds

Different model formulations (i.e., with alternative objective functions) are solved both neglecting and including LB1 and LB2, to analyze their impact on the computational times.

Eight test instances are considered, to give a wide range of situations, which have been generated as follows. The time horizon T is set either equal to 14 or 28 days, and each day is divided in |K| = 3 parts; the set B includes 8 blood types;  $d_b$  vector for the 8 blood types is assumed to be {140; 28; 56; 5; 14; 4; 140; 42} for T = 14 and {280; 56; 112; 10; 28; 9; 280; 84} for T = 28; two uncertainty values  $\varepsilon$  are chosen, i.e., 0.1 and 0.25; overtime penalty values  $\delta_k$  are selected to be 0.08, 0.06 and 0.03 for k = 1, 2, 3, respectively;  $\alpha_k$  fractions are taken equal to 0.4, 0.3 and 0.3 for k = 1, 2, 3; respectively; the number of non-booked donors  $n_t^b$  is assumed constant over the days and the values for the different blood types are set equal to {2; 0; 1; 0; 0; 0; 2; 1}; the capacity  $c_{tk}$  is assumed equal to 450 minutes for each day t and period k; visit duration r is assumed to be 20 minutes and such value is also used to compute  $R_{tk}$  when required. Maximum variation weight  $\eta$  in the objective function is finally set equal to 1.

Different values are considered for  $a_t^b$ , to evaluate both the case without  $(a_t^b = 0, \forall t, b)$ and with previously allocated slots. In the latter case, each allocated slot  $a_t^b$  is randomly generated within the 40%-80% range of the corresponding optimal values  $\xi_t^b$  of  $x_t^b$  obtained without previously booked donors  $(a_t^b = 0)$ . Moreover, the corresponding  $R_{tk}$  values are generated by randomly splitting  $a_t^b$  over the |K| periods within the day t. Names and characteristics of the instances are summarized in Table 5.

A total of 8 configurations of objective functions and LBs have been analyzed for each instance, thus obtaining 64 combinations for the pair instance-configuration. A 3600 seconds time limit and 2.5 GB memory limit have been set in all cases.

Results are reported in Tables 6–8 in terms of CPU time, objective function (OF) value, and lower bound value of the objective function (OFlow), i.e., best bound given by CPLEX.

	O	F1+OF3		OF1+O	F3 with	LB1	Time	OFlow
Inst.	CPU time	OF	OFlow	CPU time	OF	OFlow	reduction	improvement
I.1	0.18	12.14	12.14	0.17	12.14	12.14	0.01	0.00%
I.2	0.19	12.14	12.14	0.13	12.14	12.14	0.06	0.00%
I.3	16.21	10.43	10.43	0.25	10.43	10.43	15.96	0.00%
I.4	0.22	12.14	12.14	0.22	12.14	12.14	0.00	0.00%
I.5	589.80	24.43	24.43	2.21	24.43	24.43	587.59	0.00%
I.6	0.25	24.43	24.43	0.17	24.43	24.43	0.08	0.00%
I.7	$674.72^{*}$	$21.93^{*}$	$19.39^{*}$	0.29	21.93	21.93	674.43	13.10%
I.8	0.45	24.43	24.43	2.52	24.43	24.43	-2.07	0.00%

Table 6: Results for the cases OF1+OF3; \* indicates that the run is terminated because the memory limit has been reached.

	OI	F2+OF3		OF2+C	0F3 with	LB2	Time	OFlow
Inst.	CPU time	OF	OFlow	CPU time	OF	OFlow	reduction	improvement
I.1	2.15	80.00	80.00	0.29	80.00	80.00	1.86	0.00%
I.2	0.17	80.00	80.00	0.15	80.00	80.00	0.02	0.00%
I.3	1.32	72.00	72.00	0.23	72.00	72.00	1.08	0.00%
I.4	0.22	72.00	72.00	0.23	72.00	72.00	-0.01	0.00%
I.5	0.88	152.00	152.00	0.44	152.00	152.00	0.44	0.00%
I.6	0.25	152.00	152.00	0.20	152.00	152.00	0.05	0.00%
I.7	TL	136.00	128.00	0.27	136.00	136.00	3599.73	6.25%
I.8	0.22	136.00	136.00	0.22	136.00	136.00	0.00	0.00%

Table 7: Results for the cases OF2+OF3; TL indicates that the run is terminated because the time limit has been reached.

Tables also compare the case with and without LB by showing the time reduction and the per cent OFlow improvement (increase) when LB is included.

We first observe very low computational times, which increase with the number of days in T, with parameter  $\varepsilon$ , and in the absence of preallocated slots ( $a_t^b = 0 \forall t, b$ ). In particular, the absence of preallocated slots introduces symmetries among days, which result into longer computational times.

In most of the cases, the problem is solved in few seconds or even in less than 1 second. This guarantees the applicability of the model and allows refreshing the preallocation of the slots with a high frequency. Even though we suggest a daily refresh, higher frequencies are possible, which may be useful when a high number of reservation calls arrive. Longer computational times and memory or time limit stops are observed only in instance I.7. This is the most demanding instance (without previously allocated slots, with higher variability  $\varepsilon$ , and with longer horizon T); however, this instance is not realistic in the actual practice, especially for the absence of previously allocated slots.

	0	DF1+O	F2+OF	3	OI	OF1+OF2+OF			F3 with LB1 Tim		Time	OFlow	
Inst.	CPU tim	ne (	OF	OFlow	CPU	time	C	ΟF	OFl	ow	reduction	improvement	į.
I.1	14.31	9	2.14	92.14	0	.21	92	2.14	92.	14	14.10	0.00%	
I.2	0.13	9	2.14	92.14	0	.12	92	2.14	92.	14	0.01	0.00%	
I.3	531.63	8	4.86	84.86	633	8.89	84	.86	84.	86	-102.26	0.00%	
I.4	0.28 8		4.86	84.86	0	0.22 84		.86	84.	86	0.06	0.00%	
I.5	739.23	739.23 17		172.08	0	.59	170	6.43 176.43		.43	738.64	2.53%	
I.6	0.33 176.43		176.43	0	.26	170	6.43	176	.43	0.07	0.00%		
I.7	1048.19	1048.19* 162.71*		$147.08^{*}$	1530	0.07*	162	$2.71^{*}$	157.	93*	-481.88	7.37%	
I.8	90.08	16	52.71	162.71	65.31 162.		2.71 162.71		24.77	0.00%			
						(a)							_
	OF1+OF2+0			OF3 wi	th LB2		Tim	le	(	OFlow			
		Inst.	CPU	time	OF	OFlov	V	reduct	ion	imp	rovement		
		I.1	0.	17 9	92.14	92.14	1	14.1	14	(	).00%		
		I.2	0.	12 9	92.14	92.14	1	0.0	)1	(	).00%		
		I.3	43.	.56 8	84.86	84.86	5	488.0	07	(	).00%		
		I.4	0.	27 8	84.86	84.86	3	0.0	)1	(	0.00%		
		I.5	327	.76 1	76.43	176.43	3	411.4	47	4	2.53%		
		I.6	0.	24 1	76.43	176.43	3	0.0	)9	(	0.00%		
		I.7	1218	.35* 10	$52.71^{*}$	161.76	3*	-170.1	16	(	9.98%		
		I.8	97.	.71 1	62.71	162.71	1	-7.6	63	(	0.00%		
						(b)							
			OF1+	OF2+OF	3 with	LB1+LI	32	Tin	ne	(	OFlow		
	-	Inst.	CPU 1	time	OF	OFlov	V	reduc	tion	imp	rovement		
		I.1	0.2	23 9	2.14	92.14	1	14.	08		0.00%		
		I.2	0.1	12 9	2.14	92.14	1	0.0	01		0.00%		
		I.3	713.	24 8	34.86	84.86	5	-181.	61		0.00%		
		I.4	0.2	27 8	34.86	84.86	5	0.0	01		0.00%		
		I.5	0.3	33 17	76.43	176.43	3	738.	90		2.53%		
		I.6	0.2	23 17	76.43	176.43	3	0.1	10		0.00%		
		I.7	1362.	84* 16	$52.71^{*}$	157.93	3*	-314.	65		7.38%		
	-	I.8	125.	56 16	52.71	162.71	1	-35.	48		0.00%		

(c)

Table 8: Results for the cases OF1+OF2+OF3 without LBs and with LB1 (a), with LB2 (b), and with both LB1 and LB2 (c); \* indicates that the run is terminated because the memory limit has been reached.

Comparing the results with and without LB, we can see the benefit of including LBs. Even though the OFlow improvement is null in most of the cases, the CPU time reduction is generally positive and sometimes relevant. As for OF1+OF3 and OF2+OF3, the respective LB guarantees to get the optimum also in instance I.7 for which a memory or time limit is reached without LB. Moreover, the presence of LB makes all CPU times lower than 1 second. Concerning OF1+OF2+OF3, LBs generally improve the CPU times but none of the alternatives (LB1, LB2 or LB1+LB2) seems to be the best one, e.g., LB1 is the best alternative for I.5 while LB2 is the best alternative for I.3. Also, LB1+LB2 does not seem to improve the performance with respect to the single LB cases. Negative time reductions are observed for I.7 and OF1+OF2+OF3, but they are not significant because a memory limit occurred in all cases, both with and without LB.

We remind that we respect the assumptions of (11)-(12) in the 8 instances, because we are considering constant  $n_t^b$  values over t for each blood type b and because, when we include already booked donors, we are assuming that  $a_t^b > 0$  values are always lower than the  $\xi_t^b$  values of the corresponding instance with non-booked donors ( $a_t^b = 0, \forall t, b$ ). For this reason, the value of the objective function does not change within each pair of corresponding instances with non-booked and previously booked donors (e.g., I.1 and I.2).

#### 5.3. Entire approach

In this section we test the effectiveness of the entire approach on a realistic instance derived from AVIS Milan case, and we analyze the impact of the coefficients  $\lambda_d$  and  $\lambda_f$  for the prioritization policy of the allocation phase.

Experiments are conducted with a rolling approach; the preallocation model is run, at each rolling day, considering the previously assigned slots  $(a_t^b)$ , and then the newly arriving calls for reservation are addressed to one of the preallocated slots  $x_t^b$ . At the end of the day,  $a_t^b$  values are updated with the new reservations, and the day t is shifted to t + 1. Then, the two phases are repeated, and so forth. The considered rolling period consists of 200 days, and the preallocation model is run at each rolling day with a planning period of |T| = 28 horizon days. At the first rolling day, we start from an empty condition without booked donors  $(a_t^b = 0, \forall t, b)$ .

The number of donors at each rolling day and their blood types are directly taken from the historical data of AVIS Milan, considering the whole blood donations over 200 days, from April 6 to October 22, 2014. In the dataset, the daily list of donations with the associated donor ID (from which all other information can be extracted) are available. Over these days, about 51 whole blood donations were made on average per day with a total of 10124 donations. The percentages of blood groups and Rhesus factor were as follows: 33.67% for A Rh+, 5.49% for A Rh-, 10.25% for B Rh+, 1.71% for B Rh-, 3.68% for AB Rh+, 0.56%for AB Rh-, 37.60% for 0 Rh+, and 7.02% for 0 Rh-. The historical data show that the number of produced bags over these 200 days is highly variable among the days, as shown in Figure 3.

To create the instance for the test, we have simulated the subsets of booked and nonbooked donors, as the possibility of reserving a donation in AVIS Milan is quite new and no significant historical information are available. Thus, to generate the portion of booked donors, existing donors in the historical date are randomly assigned to booked or non-booked class. From a discussion with the managers of AVIS Milan, they declared that a good percentage of booked donors should be at least the 80%. Thus, each donor is independently considered to be booked with probability 0.8, and non-booked with probability 0.2.

For the non-booked donors we assume that they arrive in the same day as in the historical data. For each booked donor, we use the previous donation date and we compute the first available donation day (90 days after the previous donation for men and 180 days for women); then, date of the reservation call is generated by adding a random number of days, uniformly distributed between 0 and 30, to the first available day.

The preallocation model has been solved considering either the configuration OF1+OF3 (including LB1 in the formulation) and the configuration OF2+OF3 (including LB2 in the formulation), to evaluate the two opposite cases in terms of balancing. The following parameters have been considered: time horizon T equal to 28 days with each day divided in |K| = 3 parts; set B made of 8 blood types; overtime penalties  $\delta_k$  equal to 0.08, 0.06 and 0.03 k = 1, 2, 3, respectively; fractions  $\alpha_k$  equal to 0.4, 0.3 and 0.3 for k = 1, 2, 3, respectively; capacity  $c_{tk}$  equal to 450 minutes  $\forall t, k$ ; all visit durations equal to 20 minutes (for both r and  $R_{tk}$ ). As for  $a_t^b$  and  $R_{tk}$ , they are daily updated by the rolling approach, starting from no preassignments at the first day. Differently from Section 5.2 where the time associated with each  $a_t^b$  is randomly split among the corresponding  $R_{tk}$ , here we exactly track the assigned period k and each preallocated slot directly determines both  $a_t^b$  and  $R_{tk}$ . The remaining parameters are chosen to fit the tested case:  $d_b$  vector for the 8 blood types with |T| = 28is assumed as  $\{503; 76; 151; 22; 50; 8; 602; 98\}$  (index b follows the same order than in the sentence above where the percentages of blood groups and Rhesus factor in the historical data are listed) and the uncertainty parameter  $\varepsilon$  is taken equal to 0.25 to model the observed variability; the number of non-booked donors  $n_t^b$  is assumed to be constant over the days (no trend is observed but just noise) and the values for the different blood types are set equal to  $\{3; 1; 1; 0; 0; 0; 4; 1\}$ .

Two different configurations for the prioritization policy are considered. Either we include only the system flexibility (with  $\lambda_d = 1$  and  $\lambda_f = 0$ ) or the first available slot policy (with  $\lambda_d = 0$  and  $\lambda_f = 1$ ). Also for the two terms in the prioritization of slots, we have considered the two opposite cases to see the entire range of behaviors; any other case is intermediate among the tested combinations.

Results are separately reported in Figures 8–11 for the four tested cases. Subfigure (a) report, for the 200 rolling days, the number of donations (total number, booked and non-booked) and the  $\sum_{b} x_t^b + a_t^b$  values for the first day of the respective planning horizon (with



Figure 8: Number of donations per day for objective function OF1+OF3 with  $\lambda_d = 1$  and  $\lambda_f = 0$ : (a) total number of donations, booked donations, non-booked donations, and  $\sum_b x_1^b + a_1^b$ ; (b) comparison between the total number of donations in the test case and in the observed historical data.

Case	Mean	Minimum	Maximum
OF1+OF3 with $\lambda_d = 1$ and $\lambda_f = 0$	0.96	0	13
OF1+OF3 with $\lambda_d = 0$ and $\lambda_f = 1$	22.85	0	27
OF2+OF3 with $\lambda_d = 1$ and $\lambda_f = 0$	2.96	0	15
OF2+OF3 with $\lambda_d = 0$ and $\lambda_f = 1$	22.97	0	27

Table 9: Waiting time in days between reservation call and donation for booked donors: mean, minimum and maximum values among all booked donors in the 200 days (0 means assigned to the same day).

t = 1). Subfigures (b) report the comparison between the total number of donations in the test case and in the historical data. Moreover, the waiting times between the reservation call and the donation are reported in Table 9.

Results show that the approach is able to balance the production of blood bags among days. The part related to the booked donations, which can be optimized, is highly balanced in all of the tests. On the contrary, the part related to non-booked donations obviously fluctuates as in the historical data. Globally, comparing the outcomes with the historical data, daily fluctuations are reduced even despite the remaining 20% of uncontrolled non-booked donor arrivals. We remark that the 80% of booked donors was considered because this



Figure 9: Number of donations per day for objective function OF1+OF3 with  $\lambda_d = 0$  and  $\lambda_f = 1$ . Reported data are as in Figure 8.



Figure 10: Number of donations per day for objective function OF2+OF3 with  $\lambda_d = 1$  and  $\lambda_f = 0$ . Reported data are as in Figure 8.



Figure 11: Number of donations per day for objective function OF2+OF3 with  $\lambda_d = 0$  and  $\lambda_f = 1$ . Reported data are as in Figure 8.

represents the first goal of AVIS Milan while introducing the reservation system. However, our results show that, despite the good behavior of the approach, a remaining detriment of the balancing is present due to the 20% of non-booked donors. Thus, our suggestion is to implement all promotion policies to bring the highest number of donors to reserve the donation in advance.

Results presented above refer to all blood types together. However, a similar balancing is obtained while considering each blood type singularly. For instance, we report in Figure 12 the number of booked donations and the total number of donations, divided by blood type, for the case OF1+OF3 with  $\lambda_d = 1$  and  $\lambda_f = 0$ . Even though the variability among days is slightly higher than in the total amount of donations, the balancing is mainly guaranteed.

Comparing the different test cases, it can be seen that keeping the flexibility of the system without prioritizing the first available slot is not very effective. In fact, as shown in Table 9, waiting times between reservation call and donation are significantly longer. This has a negative impact on the amount of donations, as longer waiting times reduce the donation frequency. Moreover, without weighting the first available slot, the closest slots could remain empty, thus reducing the daily throughput of the system. On the contrary, neglecting the flexibility has not a negative impact on the outcomes. However, in our tests, we assume that donors always accept the first suggested slot (with the highest score  $S_{tkb}$ ) without evaluating donors' preferences, who might also ask to donate in a day without any empty preallocated slots. This evaluation requires data that are not included in the AVIS Milan database.

The two weights  $\lambda_d$  and  $\lambda_f$  also impact on the ramp-up period. The number of booked



Figure 12: Number of donations per day, divided by blood type, for objective function OF1+OF3 with  $\lambda_d = 1$  and  $\lambda_f = 0$ : (a) booked donations and (b) total number of donations. Labels of blood types are reported in increasing order of the associated index b.

donations does not stabilize until about the 50th day for the cases with  $\lambda_d = 0$  and  $\lambda_f = 1$ . As mentioned, flexibility spreads the donation days over the time horizon, thus letting some slots empty, while on the contrary assigning slots based only on the first available day fills the slots from early beginning, thus avoiding empty slots in the first days when the system starts with  $a_t^b = 0$ .

In all cases, after the ramp-up period,  $\sum_b x_1^b + a_1^b$  at the first day of the planning horizon is really close to the number of booked donations (equal or slightly higher). This indicates both that the  $d_b$  parameters have been appropriately set and that, once a fair prediction of  $d_b$  is considered, our system does not leave many empty preallocated slots. A slightly higher number of empty slots is present with OF1+OF3, but this amount is anyway limited.

We finally remark that the preallocation model has been always solved to the optimum in all rolling days of all the cases.

We have also considered a further experiment, besides the four tested cases, to verify that intermediate behaviors are obtained. We run the same instance with all of the terms OF1, OF2 and OF3, assuming  $\eta = 1$ ,  $\lambda_d = 5$  and  $\lambda_f = 1$ , and considering the same values for all other parameters. Actually, the presence of an intermediate behavior with respect to those above presented is confirmed.

#### 6. Discussions and conclusion

In this paper, we first define (to the best of our knowledge) and formalize the BDAS problem, and we propose an appointment scheduling framework to solve it.

Our framework for planning the assignments consists of two phases: a MILP model to preallocate time slots of the different blood types, and a prioritization policy to assign the preallocated slots. The goal is to balance the production of blood bags of each type among the days, while also avoiding physician overtimes. The main points of our framework are, besides the decomposition in two phases, the presence of both booked and non-booked donors and the degree of freedom for the number of slots to preallocate (due to the uncertainty associated with  $d_b$ ). The latter point makes our preallocation model different from the allocation and scheduling models usually available in the literature, since here the amount of entities to allocate is another decision variable, whereas it is fixed in several other cases.

The proposed approach has been successfully applied to the real case of a large blood collection centre operating in Italy, the AVIS Milan, and the results confirm the capability of the approach to balance the production of each blood type among days.

Future work will be conducted to extend the model, e.g., to include donations different than the whole blood and to consider missed donations. The latter refers to donors who reserve a donation slot but do not make the donation, because of no-show or because the physician does not admit them to donation after the visit.

Moreover, to improve the quality of the solution, we will investigate the possibility of creating a robust counterpart of the preallocation model. At present, the uncertainty is modeled through parameter  $\varepsilon$ , but the model is deterministic. On the contrary, a robust version would include uncertain parameters, at least for  $d_b$  and  $n_t^b$ .

Finally, a further extension will be the adaptation of the model to follow a predetermined demand pattern rather than a constant demand.

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